Anonymity-oriented Signatures based on Lattices

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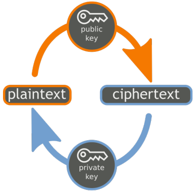
Roadmap

- Introduction
- Lattice-based cryptography and *Learning with Errors*
- Motivation for anonymity-oriented signatures
- Lattice-based group signatures
- Conclusion

Introduction

- Cryptography = design of secure protocols
  confidentiality - authenticity - integrity

- Public Key Cryptography:



  - Concept: Diffie & Hellman '76
  - The secret is secret $\rightsquigarrow$ a public key is available

$$sk \longleftrightarrow pk$$

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    - RSA '78 — factorization
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  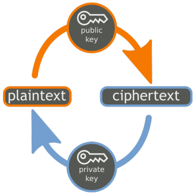

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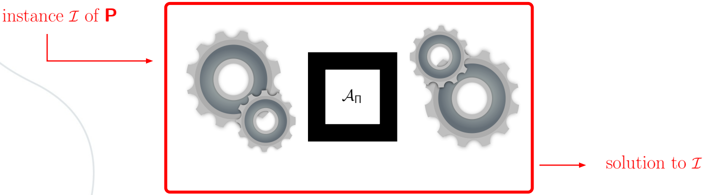
Not enough any more !

Introduction

What does secure mean ? depends on the application

- $\rightsquigarrow$ *security model* for a cryptographic primitive
- $\rightsquigarrow$ *proof* of its (in)security

  to prove = to reduce a "hard" problem **P** to an attack against the scheme **Π**



Introduction

Public-Key Cryptography:

Protocols

- e-cash
- e-voting
- anonymous access in the cloud

Cryptographic Primitives

- encryption
- signatures

Algorithmics

- hardness of arithmetic problems
- efficient operations

BE STRONG - BE QUICK - BE FUNCTIONAL

Lattice-based Cryptography and LWE

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- They must be described shortly.
  Space and communication must be low.
- The problem must be rich, flexible and expressive.
  Some applications need advanced cryptographic primitives.

Good Algorithmic Problems

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3. Instances short.
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- Security parameter λ : the best known algorithm to break the scheme must have a cost of at least 2^{λ}.
- Underlying arithmetic algorithms have a cost of $\lambda^{\mathcal{O}(1)}$.
- Instances should be represented using $\lambda^{\mathcal{O}(1)}$ bits.

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- The last criteria is less quantifiable...

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- Discrete Log and Diffie-Hellman in the group of points of a curve equipped with a pairing :

  Poor balance efficiency / security

  Richer, more flexible and expressif (e.g.. : IBE, ABE).

The *Learning With Errors* problem – **LWE**

Informally: Resolution of an overdetermined $m \times n$ linear system which is random, noisy, and modulo a short integer q.

Find $(s_1, s_2, s_3, s_4, s_5)$ such that :

$$\begin{aligned}
s_1 + 22s_2 + 17s_3 + 2s_4 + s_5 &\approx 16 \mod 23 \\
3s_1 + 2s_2 + 11s_3 + 7s_4 + 8s_5 &\approx 17 \mod 23 \\
15s_1 + 13s_2 + 10s_3 + s_4 + 22s_5 &\approx 3 \mod 23 \\
17s_1 + 11s_2 + s_3 + 10s_4 + 3s_5 &\approx 8 \mod 23 \\
2s_1 + s_2 + 13s_3 + 6s_4 + 2s_5 &\approx 9 \mod 23 \\
4s_1 + 4s_2 + s_3 + 5s_4 + s_5 &\approx 18 \mod 23 \\
11s_1 + 12s_2 + 5s_3 + s_4 + 9s_5 &\approx 7 \mod 23
\end{aligned}$$

We can have an arbitrary number of equations.

Other interpretation : decoding of a random linear code for the Euclidean distance.

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  It is often λ^2.

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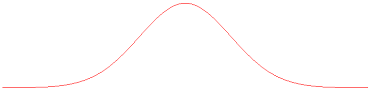
- Very rich, flexible and expressive : encryption, identity-based encryption, attribute-based encryption, homomorphic encryption, functional encryption, etc.

Gaussian Distributions

Gaussian distribution of parameter s :

$$D_s(x) \sim \frac{1}{s} \exp\left(-\pi \frac{x^2}{s^2}\right)$$
$$\forall x \in \mathbb{R}$$



Discrete Gaussian Distribution of support $\mathbb{Z}$ and of parameter s :

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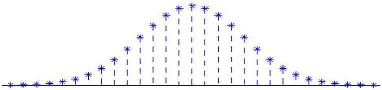

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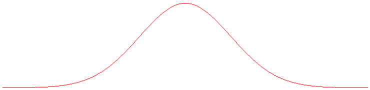

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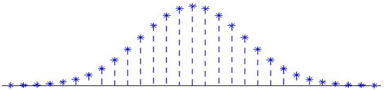


- We know how to sample efficiently.
- Most of the values are in $[-c \cdot s, +c \cdot s]$ for a constant c, if s is not too small.

The LWE problem [Regev05]

Let $n \geq 1, q \geq 2$ and $\alpha \in]0, 1[$.
For all $\mathbf{s} \in \mathbb{Z}_q^n$, let us define the distribution $D_{n,q,\alpha}(\mathbf{s})$ by :

$$(\mathbf{a}, \langle \mathbf{a}, \mathbf{s} \rangle + e)\,, \text{ avec } \mathbf{a} \hookleftarrow U(\mathbb{Z}_q^n) \text{ et } e \hookleftarrow D_{\mathbb{Z},\alpha q}.$$

Computational LWE

For all $\mathbf{s}$:

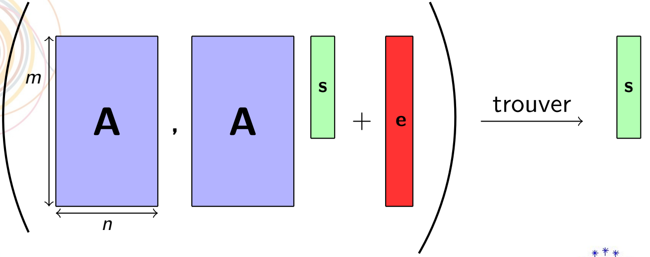
from an arbitrary number of samples of $D_{n,q,\alpha}(\mathbf{s})$, recover $\mathbf{s}$.

Decisional LWE

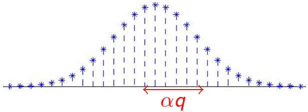
With non-negligeable probability on $\mathbf{s} \hookleftarrow U(\mathbb{Z}_q^n)$:

distinguish the two distributions $D_{n,q,\alpha}(\mathbf{s})$ and $U(\mathbb{Z}_q^{n+1})$.

LWE: matricial view



- $\mathbf{A} \hookleftarrow U(\mathbb{Z}_q^{m\times n})$,
- $\mathbf{s} \hookleftarrow U(\mathbb{Z}_q^{n})$,
- $\mathbf{e} \hookleftarrow D_{\mathbb{Z}^m,\alpha q}$.



Discrete Gaussian error

Decisional variant :
determine if $(\mathbf{A}, \mathbf{b})$ is of the form above, or uniform.

LWE: hardness

Brute Force

First variant:

- try all the possible $\mathbf{s} \in \mathbb{Z}_q^n$
- is $\mathbf{b} - \mathbf{A} \cdot \mathbf{s}$ small ?

$\Rightarrow$ Cost $\approx q^n$.

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Second variant:

- guess the n first errors.
- compute the corresponding $\mathbf{s}$.
- is $\mathbf{b} - \mathbf{A} \cdot \mathbf{s}$ small?

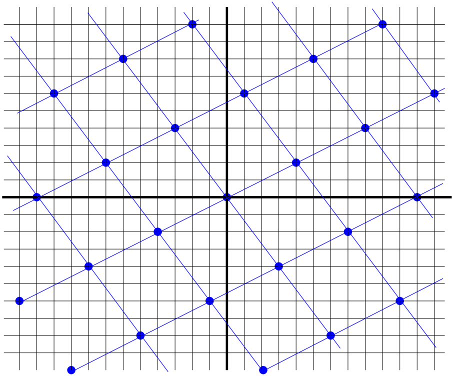
$\Rightarrow$ Cost $\approx (\alpha q \sqrt{n})^n$.

LWE and lattices

A lattice:

$$\mathcal{L} = \left\{ \sum_{i=1}^{n} x_i \mathbf{b}_i : x_i \in \mathbb{Z} \right\} \subset \mathbb{R}^n$$

If the $\mathbf{b}_i$ are linearly independant, they are called a basis.



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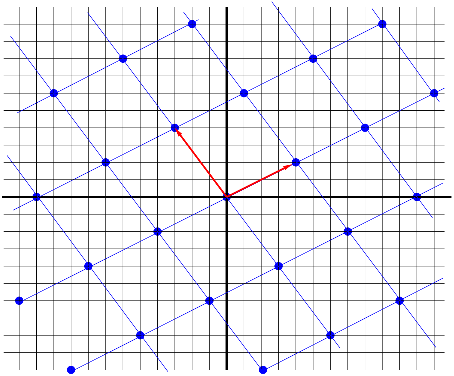

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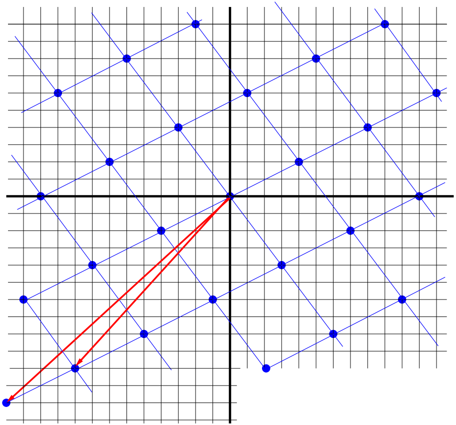


There are infinitely many basis.

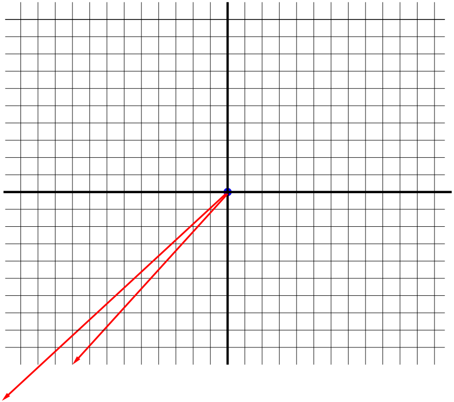
$$\begin{pmatrix} 4 & -3 \\ 2 & 4 \end{pmatrix} \cdot \underbrace{\begin{pmatrix} -4 & -3 \\ -1 & -1 \end{pmatrix}}_{\det=1} = \begin{pmatrix} -13 & -9 \\ -12 & -10 \end{pmatrix}$$

Lattices

Provide hard problems:

Shortest Vector Problem (SVP$_\gamma$)

Minimum :
$\lambda(L) = \min(\|\mathbf{b}\| : \mathbf{b} \in L \setminus \mathbf{0})$.



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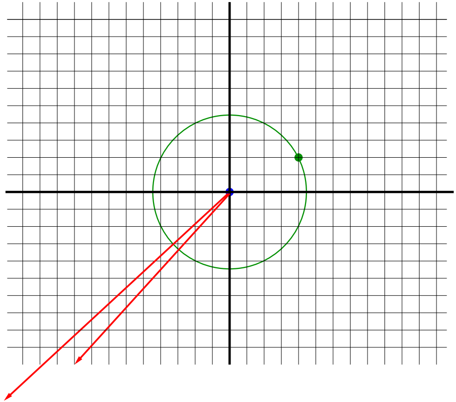

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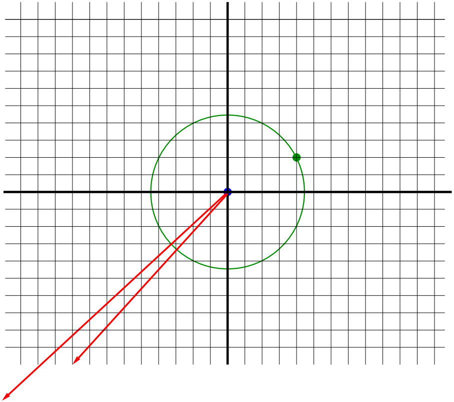

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Best known algorithm : BKZ

Time $2^t \cdot (n + \log \|B\|)^{\mathcal{O}(1)}$

$\Downarrow$

Approximation factor $\gamma \approx t^{\mathcal{O}(n/t)}$

Algorithm due to [SchnorrEuchner91], analysed by [HanrotPujolStehlé11].

Hardness of SVP

- SVP$_\gamma$: Given a basis of L, find $\mathbf{b} \in L$ s.t.

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- SVP_γ : Given a basis of L, find $\mathbf{b} \in L$ s.t.

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- GapSVP_γ: Given a basis of L and t, answer

$$\textbf{YES} \text{ if } \lambda(L) \leq t \quad \text{and} \quad \textbf{NO} \text{ if } \lambda(L) > \gamma \cdot t$$

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Hardness of GapSVP_γ

- **NP-hard** if $\gamma \leq \mathcal{O}(1)$ (probabilistic reductions) [Ajtai98,HavivRegev12]
- **in NP$\cap$coNP** if $\gamma \geq \sqrt{n}$ [GoldreichGoldwasser97,AharonovRegev05]
- **in P** si $\gamma \geq \exp\left(n \cdot \frac{\log\log n}{\log n}\right)$

LWE : difficulty

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- Solving LWE using BKZ :

  From $\mathbf{A}$ and $\mathbf{b}$, we wish to determine if $\mathbf{b}$ is an LWE sample or a uniform vector.

  Let $L = L(\boxed{\mathbf{A}}) = \{\mathbf{x} \in \mathbb{Z}^m : \mathbf{x}^T \cdot \boxed{\mathbf{A}} = \mathbf{0}^T \bmod q\}$

  - L is a lattice.
  - Its dimension is m: $q \cdot \mathbb{Z}^m \subset L$
  - Pigeonhole principle: $\lambda_1(L) \leq \sqrt{m} q^{n/m}$
  - If $\mathbf{x} \in L \setminus \mathbf{0}$ is short, then $\langle \mathbf{x}, \mathbf{b} \rangle$:
    - is small if $\mathbf{b}$ is an LWE sample because it is $\langle \mathbf{x}, \mathbf{e} \rangle$,
    - is uniforme modulo q otherwise.

  $\Rightarrow$ For the attack to work, we need

$$\|\mathbf{x}\| \alpha q \leq q \iff \|\mathbf{x}\| \leq 1/\alpha.$$

LWE : difficulty

- Decisional LWE $\iff$ Computational LWE

- Solving LWE using BKZ :
  - $\lambda_1(L) \leq \sqrt{m} q^{n/m}$.
  - We want to find $\mathbf{x} \in L$ s.t. $0 < \|\mathbf{x}\| \leq 1/\alpha$.

  In time 2^t, BKZ computes $\mathbf{x} \in L$ s.t.: $\|\mathbf{x}\| \leq t^{\mathcal{O}(m/t)} \sqrt{m} q^{n/m}$.

  The optimal m is $\approx \sqrt{tn \frac{\log q}{\log t}}$ and we get $\|\mathbf{x}\| \leq 2^{\mathcal{O}(\sqrt{\frac{n}{t} \log q \log t})}$.

BKZ's cost to break LWE

$$\text{Time: } \left(\frac{n \log q}{\log^2 \alpha}\right)^{\mathcal{O}(\frac{n \log q}{\log^2 \alpha})}.$$

LWE : difficulty

- Decisional LWE $\iff$ Computational LWE
- Solving LWE using BKZ : $\left(\frac{n \log q}{\log^2 \alpha}\right)^{\mathcal{O}\left(\frac{n \log q}{\log^2 \alpha}\right)}$
- Suppose that $\alpha q \geq 2\sqrt{n}$ and that q is prime and polynomial in n. Then there exists a quantum polynomial reduction from GapSVP_γ in dimension n to $\mathrm{LWE}_{n,q,\alpha}$, with $\gamma \approx n/\alpha$.
  [Regev05]
- There exists a classical polynomial reduction from GapSVP_γ in dimension $\approx \sqrt{n}$ to $\mathrm{LWE}_{n,q,\alpha}$, with $\gamma \approx n^2/\alpha$.
  [BrakerskiLangloisPeikertRegevStehlé13]

Regev's encryption [Regev05]

- **Parameters** : $n, m, q \in \mathbb{Z}$, $\alpha \in]0,1[$.

- **Keys** : $\mathrm{sk} = \mathbf{s}$ and $\mathrm{pk} = (\mathbf{A}, \mathbf{b})$, with $\mathbf{b} = \mathbf{A}\mathbf{s} + \mathbf{e} \bmod q$
  where $\mathbf{s} \hookleftarrow U(\mathbb{Z}_q^n)$, $\mathbf{A} \hookleftarrow U(\mathbb{Z}_q^{m \times n})$, $\mathbf{e} \hookleftarrow D_{\mathbb{Z}^m, \alpha q}$.

- **Encryption** $(M \in \{0,1\})$ **:** Let $\mathbf{r} \hookleftarrow U(\{0,1\}^m)$,

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- **Decryption** $(\mathbf{u}, v)$: Compute $v - \mathbf{u}^T \mathbf{s} \bmod q$, because:

$$\underbrace{\mathbf{r}^T \left[\mathbf{A}\,\mathbf{s} + \mathrm{e} \right] + \lfloor q/2 \rceil \cdot M}_{v} - \underbrace{\mathbf{r}^T \mathbf{A}\,\mathbf{s}}_{\mathbf{u}^T \mathbf{s}} = \text{small} + \lfloor q/2 \rceil \cdot M$$

If close to 0, output 0, else, output 1.

Correctness (probabilistic)

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Why does it work?

- We have $v - \mathbf{u}^T\mathbf{s} = \mathbf{r}^T\mathbf{e} + \lfloor q/2 \rceil \cdot M \bmod q$
- But $|\mathbf{r}^T\mathbf{e}| \leq \|\mathbf{r}\|\|\mathbf{e}\| \leq m\alpha q$, with probability ≈ 1.

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- But $|\mathbf{r}^T\mathbf{e}| \leq \|\mathbf{r}\|\|\mathbf{e}\| \leq m\alpha q$, with probability ≈ 1.
- If $M = 0$, then $v - \mathbf{u}^T\mathbf{s} \bmod q$ is at most of the order of $m\alpha q$.

  We set α so that it is $\ll q$.
- If $M = 1$, then $v - \mathbf{u}^T\mathbf{s} \bmod q$ is close to $\lfloor q/2 \rceil$.

A trapdoor for LWE

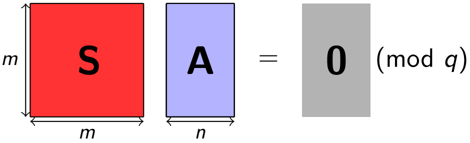
Let's recall :

$$L(\mathbf{A}) = \{\mathbf{x} \in \mathbb{Z}^m : \mathbf{x}^T \cdot \mathbf{A} = \mathbf{0}^T \bmod q\}$$

- It is a lattice of dimension m,
- A short basis allows to generate short vectors in $L(\mathbf{A})$,
- An arbitrary basis does not give any information (solution to LWE).

GenBasis : sample $\mathbf{A}$ and $\mathbf{S}$, a short basis of $L(\mathbf{A})$, simultaneously.

- $\mathbf{S} \in \mathbb{Z}^{m \times m}$ short
- We have $\mathbf{S}\,\mathbf{A} = 0 \bmod q$.
- $\mathbf{S}$ allows to invert LWE
- Can add constraints: ex. $\mathbf{B}^T \cdot \mathbf{A} = \mathbf{0}$ (with trapdoor)



Another problem

The security of our group signature also relies on :

- Short Integer Solution (SIS)

$$\boxed{\text{Given } \mathbf{A} \hookleftarrow U(\mathbb{Z}_q^{m\times n}), \text{ find } \mathbf{x} \in \mathbb{Z}^m \setminus \{\mathbf{0}\} \text{ small s.t. } \mathbf{x}^T \cdot \mathbf{A} = 0 \pmod q}$$

L., Langlois and Stehlé. *Chiffrement avancé à partir du problème Learning With Errors*. Chapitre de l'ouvrage "Informatique Mathématique, une photographie en 2014", Presses Universitaires de Perpignan (2014)

Lattice-based Cryptography Toolbox

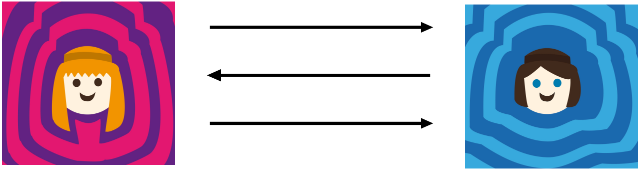
- Last tool :

  Given public $\mathbf{A} \in \mathbb{Z}_q^{m \times n}$ and $\mathbf{y} \in \mathbb{Z}_q^n$,

  there exist a (3-round) interactive protocol to prove that one knows $\mathbf{x}$ small such that

$$\mathbf{x}^T \mathbf{A} = \mathbf{y}^T$$

  without revealing any information of $\mathbf{x}$.



zero-knowledge proof of knowledge

Anonymity-Oriented Signatures

Cryptographic motivations

Need for authenticity *and* anonymity

- Anonymous credentials: anonymous use of certified attributes
  - Ex.: student card - name, picture, date, grade,...

    $\rightsquigarrow$ non-anonymous
  - Idemix (Identity-Mixer) of IBM
    Anonymous credential system developed at IBM Research [...] that enables strong authentication and privacy at the same time.

    selective revelation of attributes
- Traffic management (Vehicle Safety Communications project of the U.S. Dept. of Transportation)

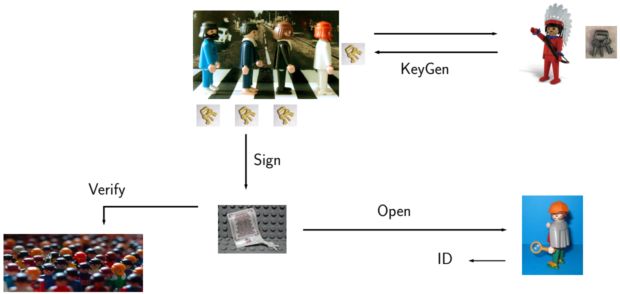
  vehicle-based collision countermeasures

Intensive use of group signatures

Group Signatures

> Group signatures allow member of a group to anonymously and accountably sign on behalf of this group

- [ChaumVanHeyst91]
- Involve :
  - Group manager $(mpk, msk) + gsk_i$ — KeyGen
  - Opening authority (osk) — Open
  - Group members (gsk_i) — Sign
  - Verify



signature $\surd$ but who signed ??

Group Signatures

Security requirements [BellareMicciancioWarinschi03] :

- Anonymity

  a given signature does not leak the identity of its originator

- Traceability

  no collusion of malicious users can produce a valid signature that cannot be traced to one of them

Issues :

- security model

  ex. anonymity

- efficiency

  compact signatures, short keys, fast operations

- additional properties

  revocation, dynamicity

Group Signatures

Generic construction [BellareMicciancioWarinschi03] :

Ingredients :

- Signature & Encryption schemes
- non-interactive zero knowledge proof system
  [FeigeLapidotShamir99] + [Sahai99] :
  if trapdoor permutations exist, then any NP-relation has a such a proof

Scheme:

- Group manager produces a certificate $Cert_i = \mathsf{Sign}_{sk_s}(i||pk_i)$
- Member i :
  1. $\sigma = \mathsf{Sign}_{\mathsf{sk}_i}(m)$
  2. $c = Encrypt_{\mathsf{pk}_o}(i||pk_i||Cert_i||\sigma)$
  3. $\Pi = \mathsf{Proof}(\sigma \text{ valid} \wedge Cert_i \text{ valid})$
  4. Output $\Sigma = (c, \Pi)$
- Verification: check the validity of proof
- Opening authority decrypts C if Π valid

Group Signatures

Security of this construction :

- It is fully-anonymous if the encryption scheme and the proof are "secure"
- It is traceable if the signature scheme and the proof are "secure"

Remarks:

- Inefficient in general
- Many constructions nevertheless follow this paradigm
- Breakthrough : [Groth06,GrothSahai2006]
  Pairing-based simulation-sound NIZK Proofs *without random oracles*

Lattice-based Group Signatures

Group Signatures with Lattices

- First lattice-based construction : [GordonKatzVaikuntanathan2010]
- Main drawbacks : size of the signatures - $O(N)$ N group members
- Ideas :
  - Keys of the authority :
    $$\begin{cases} \text{public parameters} = \{\mathbf{A}_i, \mathbf{B}_i\}_i \text{ s.t. } \mathbf{A}_i \cdot \mathbf{B}_i^T = 0 \pmod q \\ \text{tracing key} = \mathbf{S}_i \qquad \text{short basis} \\ sk_i = \mathbf{T}_i \text{ (members)} \qquad \text{short basis} \end{cases}$$
  - A signature:
    - compute short $\mathbf{e}_i$ s.t. $\mathbf{A}_i\mathbf{e}_i = H(m) \pmod q$ $(\mathbf{T}_i)$
    - $\forall j \neq i$ compute $\mathbf{e}_j$ s.t. $\mathbf{A}_j\mathbf{e}_j = H(m) \pmod q$ "pseudo-signature"
    - Encrypt each $\mathbf{e}_i$ variant of [Regev2009]
    - a proof Π disjunction of [MicciancioVadhan03]

Secure under LWE (anonymity) and GapSVP (traceability).

Group Signatures with Lattices

A new compact construction based on lattices [L.LangloisLibertStehlé2013]

Ingredients:

- [Boyen2010]'s signature (standard model)
- [GentryPeikertVaikuntanathan2008] encryption scheme

- $N = 2^\ell$ group members
- public matrices $\mathbf{A}_i$'s and $\mathbf{B}_i$'s (almost as before)
- each user is given a *short* basis $\mathbf{T}_{\mathrm{id}}$ of a public lattice associated to its identity

$$\mathbf{A}_{\mathrm{id}} = \left(\frac{\mathbf{A}}{\mathbf{A}_0 + \sum_{i=1}^{\ell} \mathrm{id}[i]\mathbf{A}_i} \right)$$

Group Signatures with Lattices

A new compact construction based on lattices [L.LangloisLibertStehlé2013]

To sign (1/3) :

- Produce $(\mathbf{x}_1 || \mathbf{x}_2)^T$ short s.t. :
$$\mathbf{x}_1{}^T \mathbf{A} + \mathbf{x}_2{}^T \cdot (\mathbf{A}_0 + \sum_{i=1}^{\ell} \mathrm{id}[i] \cdot \mathbf{A}_i) = 0 \pmod q$$

- Encrypt $\mathbf{x}_2$ as $\mathbf{c}_0 = \mathbf{B}_0 \cdot \mathbf{s}_0 + \mathbf{x}_2$ $\qquad (\mathbf{s}_0 \hookleftarrow U(\mathbb{Z}_q^n))$

  $+$ generate a proof π_0: $\mathbf{c}_0$ is close to a point in the $\mathbb{Z}_q$-span of $\mathbf{B}_0$ [Lyubashevsky2012]

- For all $i = 1, \ldots, \ell$ encrypt $\mathrm{id}_i \cdot \mathbf{x}_2$ as
$$\mathbf{c}_i = \mathbf{B}_i \cdot \mathbf{s} + p \cdot \mathbf{e}_i + \mathrm{id}_i \cdot \mathbf{x}_2$$

  so that $\begin{cases} \mathbf{c}_i \text{ and } \mathbf{c}_0 \text{ encrypt the same } \mathbf{x}_2 & (\mathrm{id}_i = 1) \\ \text{or } \mathbf{c}_i \text{ encrypts } \mathbf{0} & (\mathrm{id}_i = 0) \end{cases}$

Group Signatures with Lattices

A new compact construction based on lattices [L.LangloisLibertStehlé2013]

To sign (2/3) :

- Generate a proof $\pi_{\mathrm{OR},i}$ of these relations (disjunctions)

  [Lyubashevsky2012] for LWE + OR

- Generate a proof π_K of knowledge of the $\mathbf{e}_i$'s and $\mathrm{id}_i \cdot \mathbf{x}_2$'s with their corresponding relation

  [Lyubashevsky2012] for SIS + OR

= encode a valid Boyen's signature

Group Signatures with Lattices

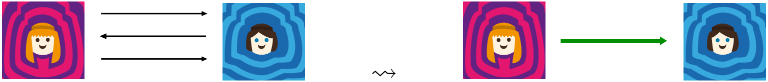
A new compact construction based on lattices [L.LangloisLibertStehlé2013]

To sign (3/3) :

What about the message ?

- interactive ZKPoK $\rightsquigarrow$ non-interactive ZKPoK *via* Fiat-Shamir

  incorporating the message in π_K



- Final signature:

$$\Sigma = \left(\{\mathbf{c}_i\}_{0 \leq i \leq \ell}, \pi_0, \{\pi_{\mathrm{OR},i}\}_{1 \leq i \leq \ell}, \pi_K \right)$$

Group Signatures with Lattices

A new compact construction based on lattices [L.LangloisLibertStehlé2013]

To Verify :

- Check the proofs

To Open :

- Decrypt $\mathbf{c}_0$ ($\rightsquigarrow \mathbf{x_2}$) and check whether $p^{-1}\mathbf{c}_i$ or $p^{-1}(\mathbf{c}_i - \mathbf{x}_2)$ is close to the $\mathbb{Z}_q$-span of $\mathbf{B}_i$.

Group Signatures with Lattices

A new compact construction based on lattices [L.LangloisLibertStehlé2013]

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- Size of the signatures : $\tilde{\mathcal{O}}(\lambda \cdot \log(N))$
- Size of the key of member i: $\tilde{\mathcal{O}}(\lambda^2)$
- Weak anonymity under LWE
- Traceability under SIS
- We provide a variant with full anonymity

Conclusion

- Anonymity-oriented signatures are useful, ex.: group signature
- Lattices are convenient to design such schemes
- Lattice-based group signatures
  - reduce the size ?
  - efficient revocation
- Lattice-based cryptography
  - competition with pairings on curves
  - functional cryptography
  - implementation
  - multi-linear maps *vs* pairings